PERGAMON

International Journal of Solids and Structures 37 (2000) 6175–6176

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Letter to the editor

Reply to Comments by X.-L. Xu on "Exact solution for the plane problem in piezoelectric materials with an elliptic or a crack", *International Journal of Solids and Structures*, Vol. 36, No. 17, pp. 2527–2540 (1999) by Cun-Fa Gao and Wei-Xun Fan

The authors wish to thank Mr Xu for his comments, which are valuable to simplify some results in the paper. The authors' responses are as follows.

(1) Eqs. (38)–(40) are valid for all transversely isotropic piezoelectric materials. However, they can be slightly simplified by letting $c_R = 0$ if one notes that $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k3}$ is a purely imaginary number, as it is found by the commentator when the properties of the materials satisfy Eq. (A6).

In fact, for any transversely isotropic piezoelectric body referred to as a Cartesian coordinate system x, y and z in which $x-y$ is the isotropic plane and the z is the poling direction, the characteristic equation of the plane problems taking place in the $x-z$ can be reduced to (Sosa, 1991; Rajapakse, 1997)

$$\mu^6 + \omega_1\mu^4 + \omega_2\mu^2 + \omega_3 = 0$$

where ω_1, ω_2 and ω_3 are positive real constants which depend on the electroelastic properties of the material.

Since ω_1, ω_2 and ω_3 are real, the solutions of the above bi-cubic equation can be written out analytically. If it is assumed that the roots of the above characteristic equation are all distinct, in general these roots take the following two forms: one is

$$\mu_1 = i\beta_1,\ \mu_2 = \alpha_2 + i\beta_2,\ \mu_3 = -\alpha_2 + i\beta_2,\ \mu_4 = \bar{\mu}_1,\ \mu_5 = \bar{\mu}_2,\ \mu_6 = \bar{\mu}_3$$

for the most media, and the other is

$$\mu_1 = i\beta_1,\ \mu_2 = i\beta_2,\ \mu_3 = i\beta_3,\ \mu_4 = \bar{\mu}_1,\ \mu_5 = \bar{\mu}_2,\ \mu_6 = \bar{\mu}_3$$

for some particular media, where β_k (k = 1, 2, 3) and α_2 are positive real numbers.

For the former, one can confirm that (A6) holds (Sosa and Khutoryansky, 1996), and then it can be shown that $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k2}$ is real and $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k3}$ is imaginary. For the latter, though (A6) is no longer valid, one can still show that $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k2}$ is real and $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k3}$ is imaginary. Thus, one has $c_R = 0$ for all transversely isotropic piezoelectric materials.

(2) The commentator thinks that some main conclusions in Gao and Fan's paper are obvious without a detailed analysis if one considers a crack filled with air or vacuum to be a slit across which both the normal components of electric displacement and the tangential component of electric field are assumed to be continuous (Parton, 1976; Parton and Kudryavtsev, 1988). This is true. In fact, recently the author used the above continuous conditions, which may be called as the Parton assumption, to study a series of two-dimensional crack problems in piezoelectric materials. The results showed that the Parton

PII: S0020-7683(99)00262-0

assumption is reliable to a mathematical crack (i.e. a crack with zero original width). Although the solutions of a mathematical crack are often used to predict the engineering fracture problems for the case of purely elastic media, such is not certainly the case for piezoelectric media, because the original width of a crack in piezoelectric solids has great effect on both the electric energy stores inside the crack and the fracture behavior of the media. Hence, one may put forward a problem: to ensure that the results based on the Parton assumption can be used as the approximate solutions of the crack problem in piezoelectric materials, what is the maximum width of a crack before deformation? It appears that some controlled experiments must be conducted to solve this problem.

(3) As pointed out by the commentator, Eq. (A6) is not valid for all cases of transversely isotropic piezoelectric materials, for example, for those where the characteristic equation has only purely imaginary roots (Rajapakse, 1997). However, this does not affect the results of the paper, because $\Sigma_{k=1}^{3}\kappa_k\Lambda_{k2}$ is still real even for the particular cases. In addition, though the analysis procedure of the paper is not related to the introduction of Eq. (A6), the authors think that (A6) is necessary to obtain a better understanding of the mechanical–electric couple relations taking place in transversely isotropic piezoelectric solids. For example, (44) can be simplified to (45) by using (A6), and as a result one can find from (45) and (53) that the electric field inside the crack and the singularity of D_2 in the materials are independent of σ_{12}^{∞}. However, this is not true for the cases of general anisotropic piezoelectric materials (Gao and Wang, 1999).

References

Gao, C.F., Wang, M.Z., 1999. Collinear permeable cracks between dissimilar piezoelectric materials, International Journal of Solids and Structures (in press).

Parton, V.Z., 1976. Fracture mechanics of piezoelectric materials. Acta Astronautica 3, 671–683.

Parton, V.Z., Kudryavtsev, B.A., 1988. Electromagnetoelasticity. Gordon & Breach, New York.

Rajapakse, R.K.N.D., 1997. Plane strain/stress solutions for piezoelectric solids. Composites 28B, 385–396.

Sosa, H.A., 1991. Plane problems in piezoelectric media with defects. International Journal of Solids and Structures 28, 491–505.

Sosa, H.A., Khutoryansky, N., 1996. New developments concerning piezoelectric materials with defects. International Journal of Solids and Structures 33, 3399–3414.

Cun-Fa Gao[a]*
Wei-Xun Fan[b]
[a]*Department of Mechanics and Engineering Science, Peking University, Beijing, 100871 People's Republic of China*
[b]*Department of Aircraft, Nanjing University of Aeronautics & Astronautics, Nanjing, 210016 People's Republic of China*

1 July 1999

* Corresponding author.